

W = emgr(f,g,s,t,w,pr,nf,ut,us,xs,um,xm,dp)

emgr – Empirical Gramian Framework (Version 5.7)

Mandatory Arguments

f	System Vector Field	(Handle) $\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, \mathbf{t})$	i.e.: $\mathbf{f} = @(x,u,p,t) \ A*x+B*u+F*p$
g	Output Functional	(Handle) $\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{p}, \mathbf{t})$	i.e.: $\mathbf{g} = @(x,u,p,t) \ C*x+D*u$ $\mathbf{1} \quad \mathbf{y} = \mathbf{x}$
s	System Dimensions	(Vector) $\mathbf{s} = [\mathbf{M}, \mathbf{N}, \mathbf{Q}]$	(Inputs, States, Outputs)
t	Time Discretization	(Vector) $\mathbf{t} = [\mathbf{dt}, \mathbf{Tf}]$	(Time Step, Time Horizon)
w	Gramian Type	(Char)	Empirical System Gramian Type 'c' Empirical Controllability Gramian (returns \mathbf{W}_c) 'o' Empirical Observability Gramian (returns \mathbf{W}_o) 'x' Empirical Cross Gramian (returns \mathbf{W}_x) 'y' Empirical Linear Cross Gramian (returns \mathbf{W}_y) 's' Empirical Sensitivity Gramian (returns $\mathbf{W}_c, \mathbf{W}_s$) 'i' Empirical Identifiability Gramian (returns $\mathbf{W}_o, \mathbf{W}_i$) 'j' Empirical Joint Gramian (returns $\mathbf{W}_x, \mathbf{W}_j$)

Optional Arguments

pr	Parameters	(Vector) Column vector of parameters (default: $\mathbf{pr} = \mathbf{0}$) (Matrix) Set of parameter columns ($\mathbf{W}_s, \mathbf{W}_i, \mathbf{W}_j$ require min / max)
nf	Options Flags	(Vector) Twelve components (default: $\mathbf{nf} = \mathbf{0}$)
ut	Input Function	(Handle) Input function $\mathbf{u}_t = \mathbf{ut}(\mathbf{t})$ or char (default: $\mathbf{ut} = 'i'$) 'i' Delta impulse (default) 's' Step input 'c' Decaying exponential chirp 'r' Pseudo-random binary
us	Steady-State Input	(Scalar) Uniform steady-state input (default: $\mathbf{us} = \mathbf{0}$) (Vector) Individual steady-state input ($\mathbf{M} \times \mathbf{1}$)
xs	Steady-State	(Scalar) Uniform steady-state (default: $\mathbf{xs} = \mathbf{0}$) (Vector) Individual steady-states ($\mathbf{N} \times \mathbf{1}$)
um	Input Scales	(Scalar) Uniform max input scales (default: $\mathbf{um} = \mathbf{1}$) (Vector) Individual max input scales ($\mathbf{M} \times \mathbf{1}$) (Matrix) Custom input scales ($\mathbf{M} \times *$)
xm	Steady-State Scales	(Scalar) Uniform max steady-state scales (default: $\mathbf{xm} = \mathbf{1}$) (Vector) Individual max steady-state scales ($\mathbf{N} \times \mathbf{1}$) (Matrix) Custom steady-state scales ($\mathbf{N} \times *$)
dp	Dot Product	(Handle) Handle to custom inner product $\mathbf{xy} = \mathbf{dp}(\mathbf{x}, \mathbf{y})$ [] Default matrix product

Option Flags

nf(1)	Trajectory centering	0 None (default) 1 Initial state 2 Final steady-state 3 Arithmetic average 4 Root-mean-squared 5 Midrange 6 Geometric mean
nf(2)	Input scale sequence	0 Single (default) 1 Linear 2 Geometric 3 Logarithmic 4 Sparse
nf(3)	State scale sequence	0 Single (default) 1 Linear 2 Geometric 3 Logarithmic 4 Sparse
nf(4)	Input transformation	0 \pm Unit (default) 1 + Unit
nf(5)	State transformation	0 \pm Unit (default) 1 + Unit
nf(6)	Normalizing	0 None (default) 1 Jacobi 2 Steady-state
nf(7)	State Gramian Type ($\mathbf{W}_o, \mathbf{W}_x, \mathbf{W}_y, \mathbf{W}_i, \mathbf{W}_j$ only)	0 Regular (default) 1 Non-symmetric Cross Gramian 2 Averaged Observability Gramian
nf(8)	Extra input ($\mathbf{W}_o, \mathbf{W}_x, \mathbf{W}_s, \mathbf{W}_i, \mathbf{W}_j$ only)	0 No (default) 1 Yes
nf(9)	Center param. Scales ($\mathbf{W}_s, \mathbf{W}_i, \mathbf{W}_j$ only)	0 No centering (default) 1 Linear mean centering 2 Logarithmic mean centering
nf(10)	Parameter Gramian Type ($\mathbf{W}_s, \mathbf{W}_i, \mathbf{W}_j$ only)	\mathbf{W}_s 0 Input-state average (default) \mathbf{W}_s 1 Input-output average $\mathbf{W}_i, \mathbf{W}_j$ 0 Detailed Schur-complement (default) $\mathbf{W}_i, \mathbf{W}_j$ 1 Approximate Schur-complement
nf(11)	Partitioned cross Gramian ($\mathbf{W}_x, \mathbf{W}_j$ only)	0 Full cross Gramian <N Cross Gramian partition size
nf(12)	Partitioned cross Gramian ($\mathbf{W}_x, \mathbf{W}_j$ only)	0 Full cross Gramian >0 Partition running index

Custom Solver

Global variable **ODE** to a handle with signature:
 $\mathbf{y} = \mathbf{ODE}(\mathbf{f}, \mathbf{g}, \mathbf{t}, \mathbf{x0}, \mathbf{u}, \mathbf{p})$ Default: RK – SSP32

Minimal Usage: $\mathbf{W} = \mathbf{emgr}(\mathbf{f}, \mathbf{g}, \mathbf{s}, \mathbf{t}, \mathbf{w})$

About Info: $\mathbf{V} = \mathbf{emgr}('version')$

More info at: <https://gramian.de>